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IV.

NOTE ON AN APPROXIMATE TRIGONOMETRIC EXPRESSION FOR THE FLUCTUATIONS OF STEAM TEMPERATURE IN AN ENGINE CYLINDER.

By Edwin H. Hall.

Presented January 11, 1893.

During one of the long interruptions of an experimental investigation of cylinder condensation in steam-engines,* I was moved to attack the problem mathematically, making use of the well known methods of treating heat conduction, which are set forth to advantage in Riemann's *Partielle Differentialgleichungen.* I carried the process far enough to see that the waves of heat produced at the inner surface of the cylinder wall, by the fluctuations of steam temperature during each stroke of the piston, would penetrate to a considerable depth; but the labor of a complete investigation of the problem by this method seemed likely to be much greater than that required by the experimental method, while the results would necessarily be open to considerable doubt, owing to the assumption that one must make in estimating the temperature of the inner surface of the cylinder wall at any point of the stroke. I therefore returned to my experiments, convinced that experiment alone could give the results for which I was striving. Nevertheless, it may be of interest to those concerned with problems of heat conduction, theoretical or practical, to see the result of the first mathematical steps, the derivation of an approximate trigonometric expression for the periodic changes of temperature suffered by the steam during each complete stroke of the piston.

The indicator diagram, Figure 1, gave the pressure, and so the temperature, of the steam at every point of the stroke. Assuming the engine piston to have simple harmonic motion, that is, neglecting the effect of the *vertical* movement of the crank pin, I plotted a curve, the heavy line of Figure 2, ordinates representing temperature in excess of 100° C., and abscissas representing phases of the stroke in terms of the angular distance of the crank pin from its zero position, the position occupied at the end of the out stroke. The left end of the base line indicates $-\pi$, or $-180°$, the beginning of forward stroke; the middle of the line represents 0°; the right end represents $+\pi$, or $+180°$, the end of back stroke. Thus the cycle of a complete forward and back stroke is compassed, and the length of the base line is called 2π.

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* See *ante*, page 37.

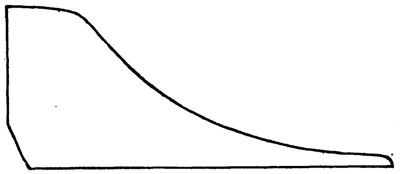

Fig. 1.

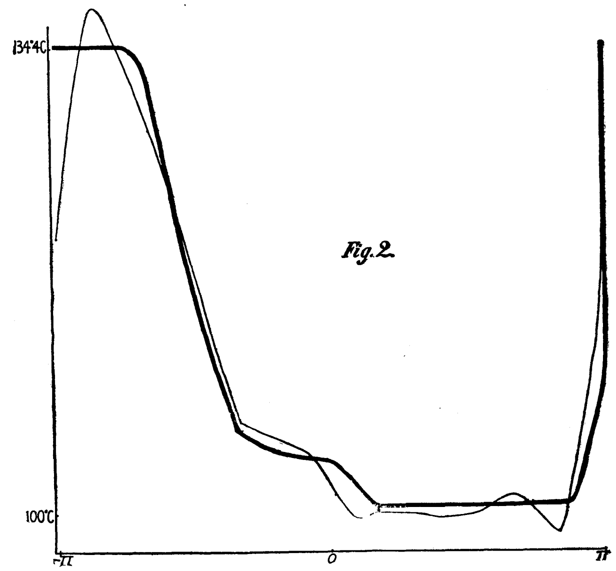

Fig. 2.

My task now was to find a mathematical expression for any ordinate, y, of the given curve in terms of the corresponding abscissa, x; that is, to write $y = \phi(x)$, and find the form of the function ϕ.

It is well known that for any continuous curve, however irregular, extending from $x = -\pi$ to $x = \pi$, the relation of any y to the corresponding x may be written as follows: —

$$y = \phi(x) = \tfrac{1}{2} b_0 + b_1 \cos x + b_2 \cos 2x + \ldots + b_m \cos m x + \ldots$$
$$+ a_1 \sin x + a_2 \sin 2x + \ldots + a_m \sin m x + \ldots$$

The length of this series of terms may be, and usually is, infinite, but frequently a comparatively small number of terms will express the required relation with sufficient accuracy, when the values of the coefficients b_0, b_1, a_1, etc. are known. I used eleven terms, six containing b coefficients and five containing a coefficients, and the labor of determining the coefficients was considerable. The general expression for any b coefficient is

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} \phi(a) \cos m a \, d a,$$

and for any a coefficient

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} \phi(a) \sin m a \, d a,$$

in which expressions a is any variable beginning with the value $-\pi$, and increasing regularly to $+\pi$, and $\phi(a)$ bears to a the same relation that $\phi(x)$ bears to x. The process of obtaining the value of any coefficient is mainly graphical. To illustrate, let us take the case of b_2. The value of $\cos 2a$ was found for fifteen values of a, beginning with $a = -\pi$, and ending with $a = +\pi$. The corresponding values of $\phi(a)$ were obtained from the heavy curve already described in Figure 2 by measurement of the ordinates corresponding to the chosen values of a. Then the product $\phi(a) \cos 2a$ was taken for each of the chosen values of a, and, a new base line extending from $-\pi$ to $+\pi$ having been laid off, these products gave the values of the ordinates at the chosen fifteen points along this base line. Through the tops of the ordinates a curve was drawn, and the area between this curve and the base line was measured by means of a planimeter. This area gave the value of the definite integral

$$\int_{-\pi}^{\pi} \phi(a) \cos 2a \, d a.$$

The values thus found for the various coefficients were as follows: —

	For Fahr. Scale.	For Cent. Scale.	For Fahr. Scale.	For Cent. Scale.
b_0	35	19.3+		
b_1	–16.9	–9.4	$a_1 =$ –20.7	–11.5
b_2	1.9	1.1–	$a_2 =$ 14.1	7.8+
b_3	1.7	1.0–	$a_3 =$ –7.6	–4.2+
b_4	0.4	0.2+	$a_4 =$ +3.1	1.7+
b_5	–1.4	–0.8–	$a_5 =$ –3.2	–1.8–

The faint line of Figure 2 shows the curve really represented by the formula $y = \phi(x)$. As every new coefficient requires, in general, more work for its determination than those preceding, I attempted no closer approximation to the actual curve of the steam temperatures than is shown in this diagram.